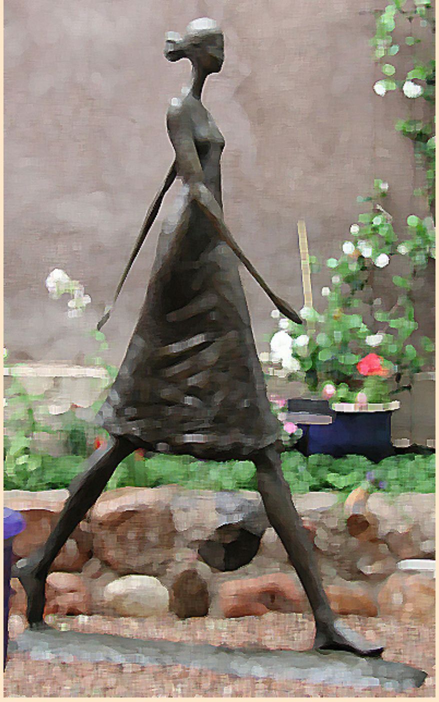


Introduction to graphs and trajectories

Sample Modelling Activities with Excel and Modellus

ITforUS

(Information Technology for Understanding Science)

Education and Culture Socrates

IT for US
Information Technology for Understanding Science
Socrates Comenius programme
Action 2.1 - Training of School Education Staff
119001-CP-1-2004-1-PL-COMENIUS-C21

I. Introduction

This module illustrate activities related to the interpretation of graphs, tables and functions in the context of motion, using Modellus and a spreadsheet.

1. Background Theory

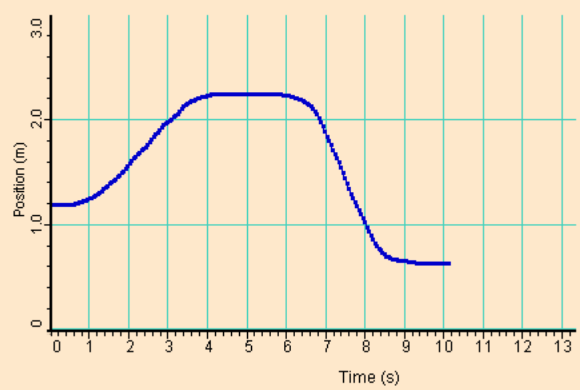
Graphs

Graphs, tables and functions are an essential part of the language of science. A **graph** is a **pictorial representation of experimental data** or of a **functional relationship between two or more variables**. Graphs can easily show general tendencies in data but can also be inaccurate or misleading, particularly graphs of variables that are functions of time: **readers tend to consider *graph shapes* as "trajectories in space" instead of relations between physical quantities**, such as distance to a sensor (or speed) and time elapsed. This misconception has been verified in many studies (see, e.g. the 2001 OECD report *Knowledge and Skills for Life*, available in http://www.pisa.oecd.org).

The use of computers provides an instructional approach that cannot be matched by non technological instruction since gives the student the **opportunity to see graphs develop in real time** and to **explore multiple representations simultaneously** (graphs, algebraic, trajectories, tables).

Motion sensors and real time graphing is becoming a standard technique for teaching graphs in Physics and in Mathematics. In this module, **the mouse is used as a motion sensor**, allowing the student or the teacher to control the motion more easily, in one dimension or in two dimensions.

Some of the activities also involve the **analysis of graphs obtained with a motion sensor** based on the reflection of ultrasound. These graphs are then used to create mathematical models, using functions.



An example of a graph that represents the motion of a walking person.
On the horizontal axis, elapsed time in seconds, started at an arbitrary instant (0 s on the graph) at a distance to the sensor of 1.2 m.
On the vertical axis, distance to the sensor, measured in meters. It is assumed that the motion is in a straight line, but that is not allaways true since the person can move laterally and the sensor still can detect the person.
During 10 s, the elapsed distance was approximately (2.2 m – 1.2 m) + (2.2 m – 0.6 m) = 1.0 m + 1.6 m = 2.6 m. Between 0.5 s and 4.0 s, the person drove away from the sensor and between 6.0 s and 9.0 s come near to the sensor.

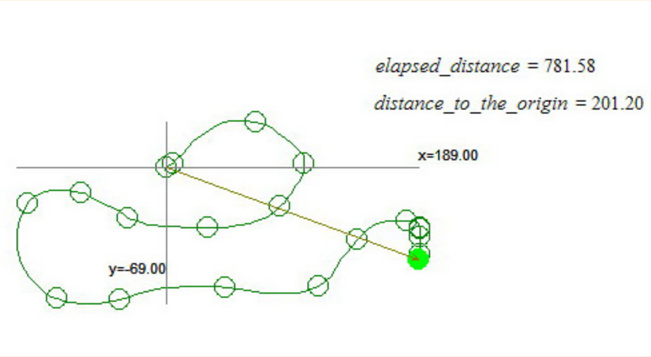
Basic motion concepts

The mathematical description of motion is not an easy task. It is common to find in books and software terms that are misleading and can induce misconceptions. For example, most data logging software use the word "position" to refer to the distance to the motion sensor. In this activity, we distinguish between **position** (a point in space described by **coordinates** in a certain reference frame), **distance to the sensor** (a quantity that is always positive) and **elapsed distance** (a quantity that is also always positive).

The module also assumes that the user is familiar wtih the **distintion between scalar and vector quantities**. Scalars are quantities expressed by only one number and vectors are quantities expressed by more than one number. For example, distance to the origin and elapse distance are scalars but position is a vector. Position, to be described in a plane, needs two numbers or coordinates. And, in space, it needs three numbers/coordinates.

A vector quantity has **magnitude** and **direction**. Both magnitude and direction can be computed from the vector components or the vector components can be computed from magnitude and direction.

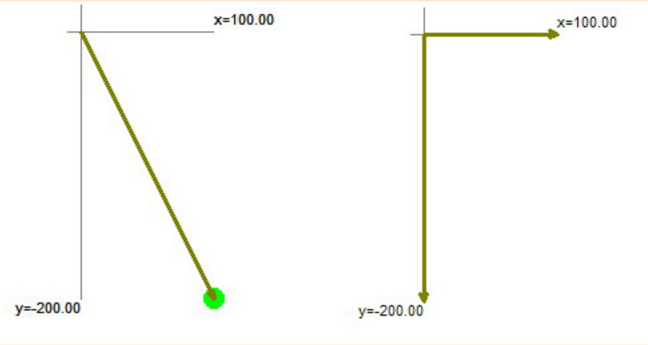
Velocity, acceleration and force, as well as position, are vector quantitites. The **magnitude of velocity** is called **speed**.



A trajectory of a particle in a plane, made with Modellus. In pixels, the distance to the origin, is

$$\sqrt{189^2 + (-69)^2} = 201.20 \ .$$

The elapsed distance is the measure on the lenght of the trajectory.



The position vector of a point in a plane with coordinates, in pixels, x = 100 and y = − 200.
A vector can be computed as the sum of its **vector components** along each axis.
For each vector component, there is a **scalar component**, with a value equal to the coordinates of the vector head if the tail of the vector is in the origin of the reference frame.
The magnitude of the above vector is

$$\sqrt{100^2 + (-200)^2} = 223.61$$

and its horizontal and vertical scalar components are 100 pixels and − 200 pixels, respectively.
The angle of vector with the horizontal axis is

$$\arctan \frac{200}{100} = 63.43^{o} \ .$$

Knowing the scalar components, we can compute the direction to where the vector points and its magnitude. Or, inversely, knowing the direction and the magnitude, compute the scalar components.

2 Science concepts introduced in this module

This module uses concepts from Physics and Mathematics but is not suitable to introduce most of them for the first time. The goal of the module is to **illustrate in a concrete way concepts that students have been taught previously**, such as:

- Reference frame;
- Origin of a reference frame and coordinates in a plane;
- Independent and dependent variables;
- Functions;
- Scales;
- Graph of a function;
- Speed and velocity;
- Trajectory;
- Linear, quadratic and sinusoidal functions.

3. Other information

It is possible to find interactive activities on the Internet about motion, graphs and trajectories. The site **"The moving man"** (http://www.mste.uiuc.edu/Murphy/MovingMan/MovingMan.html), used for research study on graph interpretation, has a interesting applet that can be used with all type of students.

Robert J. Beichner has done **pioneer research and development** on graph skills. Most of its papers can be found at http://www2.ncsu.edu/ncsu/pams/physics/People/beichner.html.

Lillian C. McDermott and Edward F. Redish published in 1999 a "Resource Letter" that **synthesizes physics education research** in the 20th century, including relevant references to graph understanding. It can be read at http://www.phys.washington.edu/groups/peg/rl.htm.

II. Didactical approach

1. Pedagogical context

The activities presented in this module can be used with students of different ages, **starting from 12 or 13 years until upper secondary school**, either in Physics or in Mathematics classes.

They were not designed to fit in any curriculum. They simple illustrate how two interactive computer tools (Modellus and a spreadsheet like Excel) can be used to improve the teaching of graphs. They can be **particularly useful for simultaneous training of Physics and Mathematics teachers**, promoting interdisciplinarity and reflection about concepts and representations.

2. Common student difficulties

Physics education has consistently verified that very often students have very severe difficulties in the construction of line graphs as well on their interpretation. As mentioned above, the most common observed misinterpretation is that **a large proportion of students view line graphs as paths or trajectories of motion**, not as kinematics quantities that are represented as functions of time. Other common difficulties include understanding slopes and rates of change.

A bibliography about learning graph skills

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Joint Matriculation Board, & Shell Centre for Mathematics Education. (1985). *The Language of Functions and Graphs, An Examination Module for Secondary Schools*.

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Rogers, L. T. (1995). The computer as an aid for exploring graphs. *School Science Review, 76*(276), 31-39.

Testa, I., Monroy, G., & Sassi, E. (2002). Students' reading images in kinematics: the case of real-time graphs. *International Journal of Science Education, 24*(3), 235-256.

3. Evaluation of ICT

Computers are now the most common scientific tool, used in almost all aspects of the scientific endeavour, from measuring and modelling to writing and synchronous communication. It should then be *natural* to use computers in learning science.

Computers can be particular useful for learning **dynamic representations**, such as graphs and functions, because they allow the user to **explore multiple representations simultaneously**. But this *is not necessarily a factor of success in learning because learners can become confused with too many simultaneous representations*. **Careful teacher guidance** is essential to sense making of multiple representations: learners need to be guided in the process of **verbalization** of visual and algebraic representations and in the process of **linking multiple representations** of the same phenomenon.

4. Teaching approaches

Good classroom organization is an essential component in a successful teaching approach, particularly when using complex tools such as computers and software. Most approaches to classroom organization that can give good results **mix features** of **students' autonomous work**, both individually and in small groups, to **teacher lecturing** to all class.

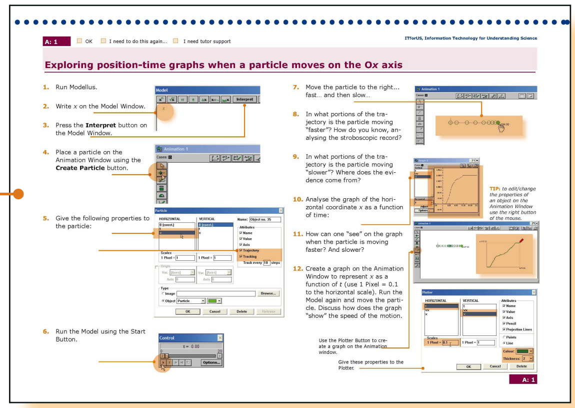
Typically, teachers can start with an all class approach, with students following the lesson with a screen projector. It is almost always a good idea to *ask one or more students to work on the computer connected to the projector*. This allows the teacher to have direct information of students' difficulties when manipulating the software and to be slower on the explanation of the ideas and activities that are being presented.

As all teachers know by experience, it is usually difficult for most students to follow written instructions, even when these instructions are only a few sentences long. To overcome this difficulty, teachers can ask students to **read the activities before** starting them and then promote a collective or group **discussion about what is supposed to be done with the computer**. *As a rule of thumb, students should only start an activity when they know what they will do on the activity: they will only consult the written worksheet just for checking details, not for following instructions.*

III. Activities

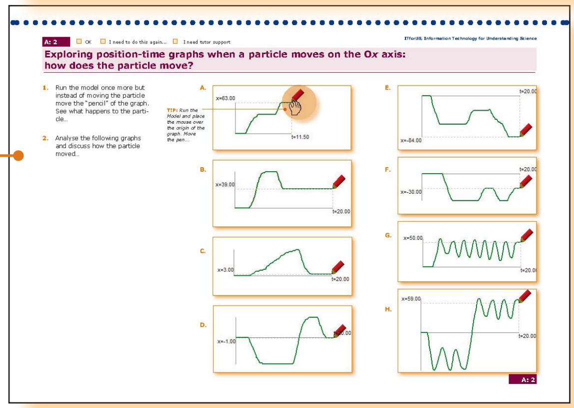
Exploring position-time graphs when a particle moves on one axis (*x* or *y*)

Basic instructions on how to:



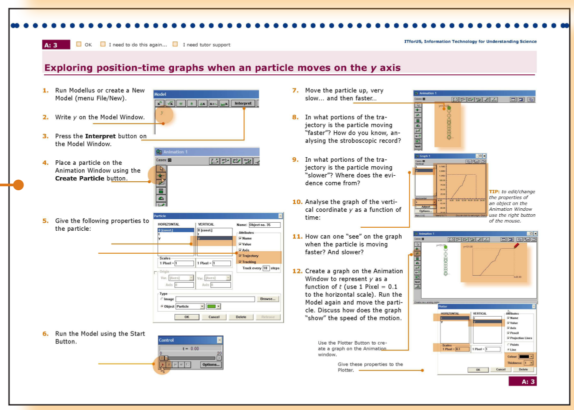
1. create a variable for a coordinate *x* of a particle;
2. place the particle on the Animation Window with *x* as the horizontal coordinate;
3. create a graph of *x* as a function of time *t* with adequate scales;
4. run the model and explore the graph of *x* in real time.

Making different graphs...

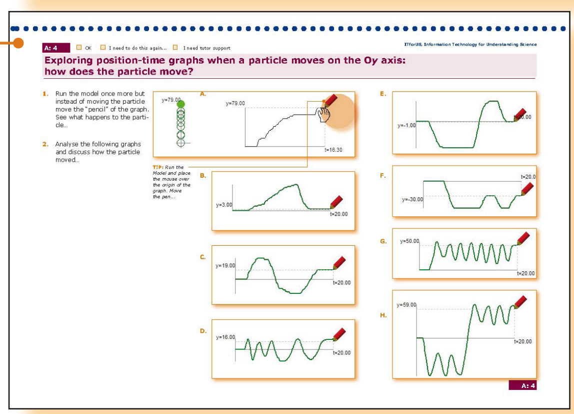


- particle moving to the right...
- stopping...
- moving again...
- moving slower and faster...
- moving to the left...
- oscillating...
- etc.

A similar activity, but with the particle moving on a vertical Oy axis...



Making different graphs with the particle moving on the vertical axis...



- particle moving up...
- stopping...
- moving again...
- moving slower and faster...
- moving and down...
- oscillating...
- etc.

Exploring position-time graphs with linear functions

This set of activities assumes that the user has a basic knowledge of what is a **linear function** and that it can represent a motion with **constant velocity**.

Basic instructions on how to:

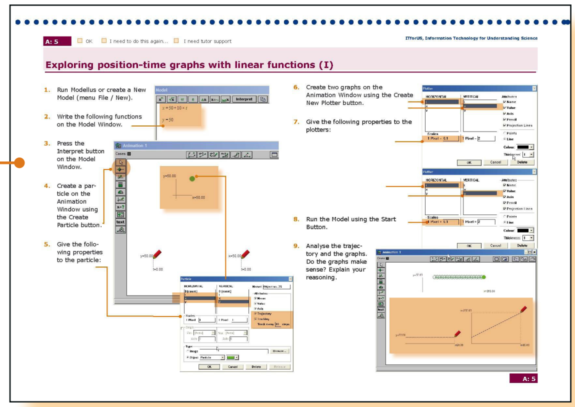
1. define functions to describe coordinates *x* and *y* of a particle;
2. place the particle on the Animation Window with *x* as the horizontal coordinate and *y* as the vertical coordinate;
3. create a graph of *x* as a function of time *t* with adequate scales;
4. create a graph of *y* as a function of time *t* with adequate scales;
5. run the model and see the graphs of *x* and *y* in real time.

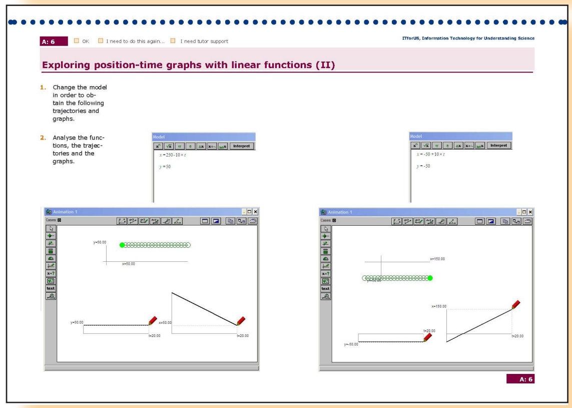
The particle can move:

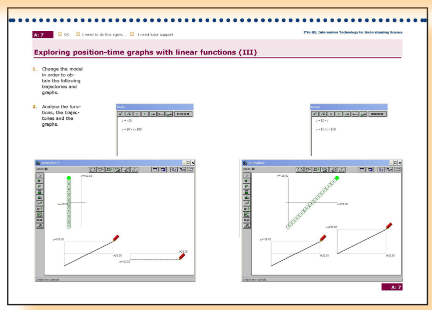
- only on the horizontal axis...
- only on the vertical axis...
- on both the horizontal and the vertical axes...
- from left to right...
- from right to left...
- from the bottom to the top...
- from the top to the bottom...
- etc.

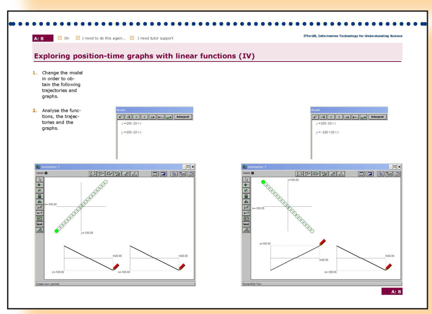
Particular emphasis should be given to differentiate between the trajectory (the path followed by an object moving through space) and the graphs.

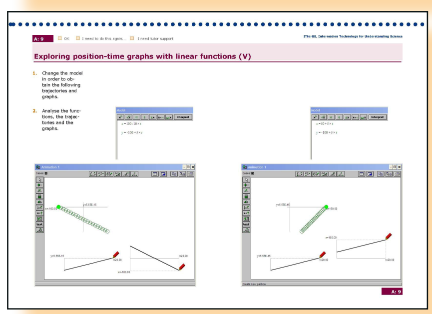
The trajectory refers to position in space and the graphs of *x* and *y* as functions of time refer to the value of a physical quantity (the value of a coordinate) in each instant of time in a certain time interval.







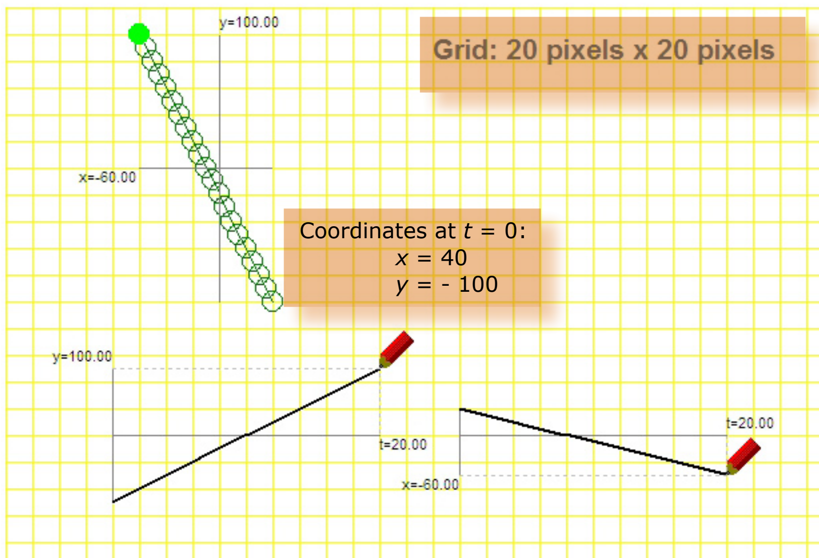




Can you deduce the functions from the graphs?

These two activities illustrate a different approach to the relation between trajectory and position-time graphs: students must now analyse trajectories and graphs and **find the algebraic expressions** of the functions that describe them.

An example of the reasoning expected from students (they should be *encouraged to use a trial and error approach*):



Slope of the y coordinate:

$$\frac{100-(-100)}{20}=\frac{200}{20}=20$$

Function:

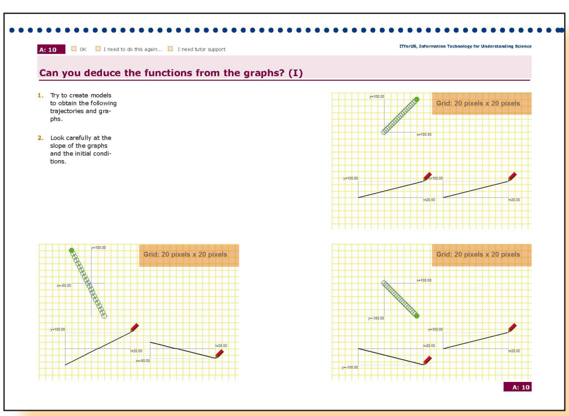
$$y=20t-100$$

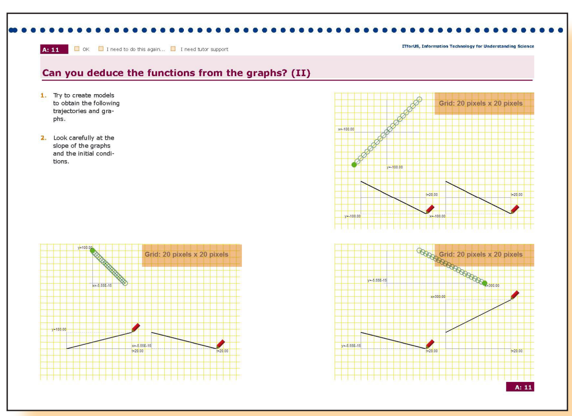
Slope of the x coordinate:

$$\frac{-60-(40)}{20}=\frac{-100}{20}=-10$$

Function:

$$x=-10t+40$$



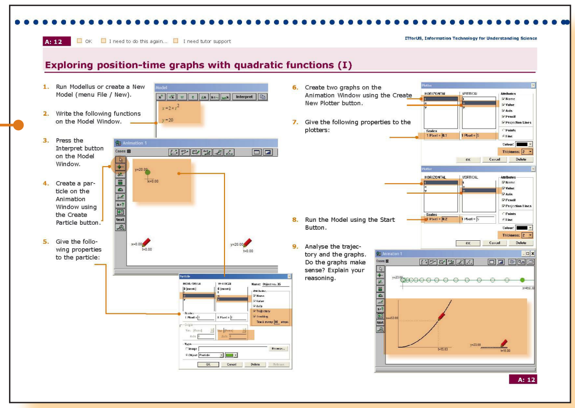


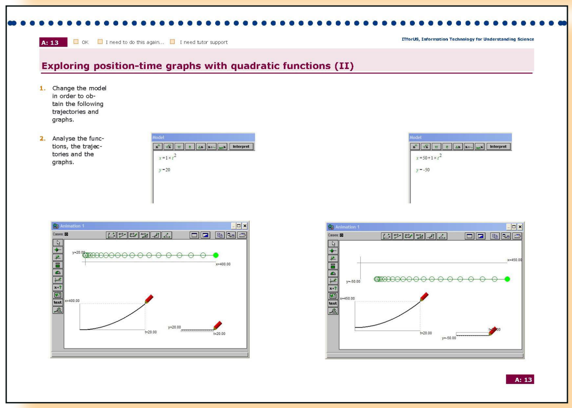
Exploring position-time graphs with quadratic functions

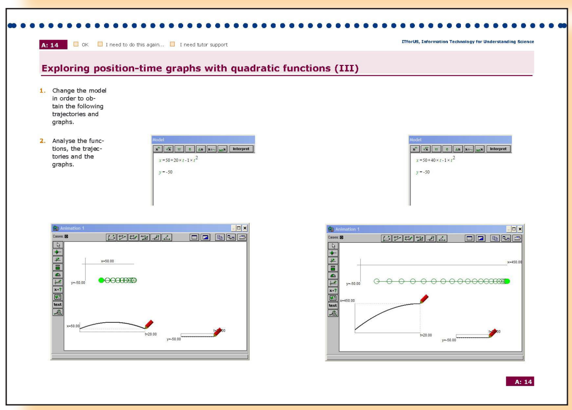
This set of activities assumes that the user has a basic knowledge of what is a **quadratic function** and that it can represent a **motion with constant acceleration**.

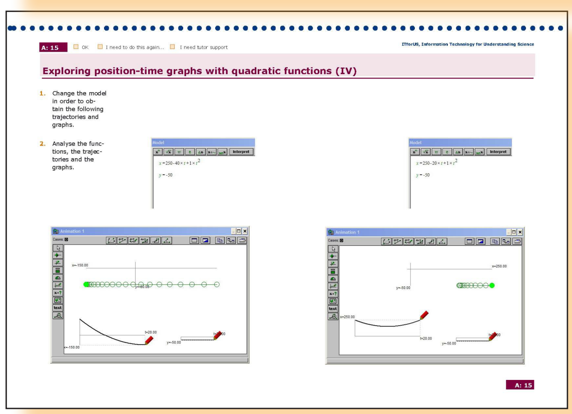
Basic instructions on how to:

1. define functions to describe coordinates *x* and *y* of a particle;
2. place the particle on the Animation Window with *x* as the horizontal coordinate and *y* as the vertical coordinate;
3. create a graph of *x* as a function of time *t* with adequate scales;
4. create a graph of *y* as a function of time *t* with adequate scales;
5. run the model and see the graphs of *x* and *y* in real time.





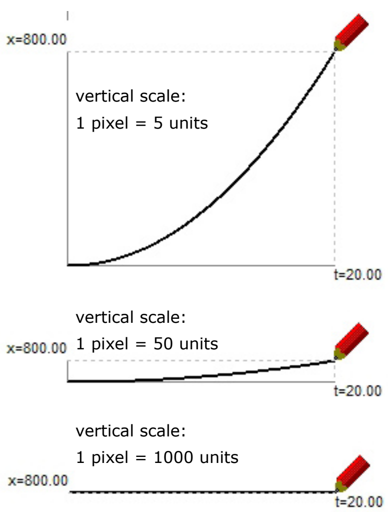




The particle is set to move only on the horizontal axis but the model could be easily changed to make the particle also move on another axis. A **further exploration of this activity could include the motion of projectiles**.

It can also be useful to **explore the same graph in different scales**, including scales where the curve seems to be a straight line (see the example below)!)

These three graphs show the same function with different vertical scales...

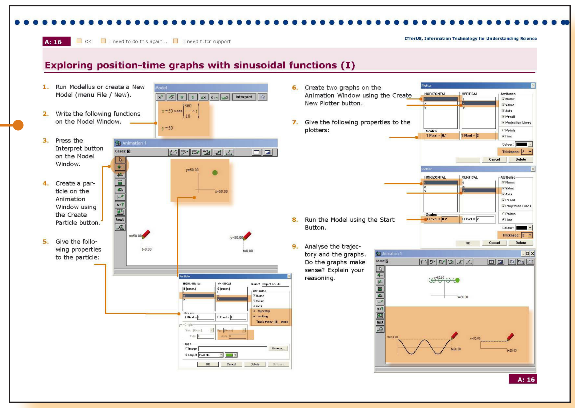


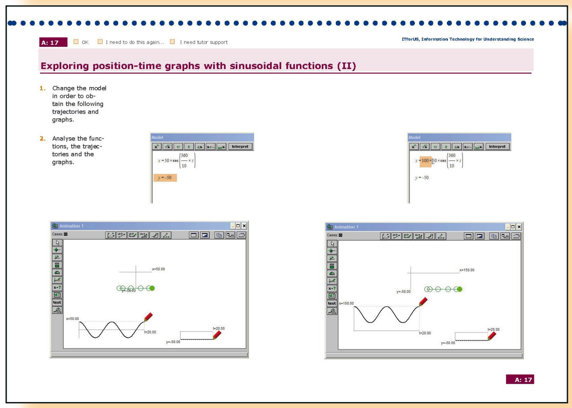
Exploring position-time graphs with sinusoidal functions

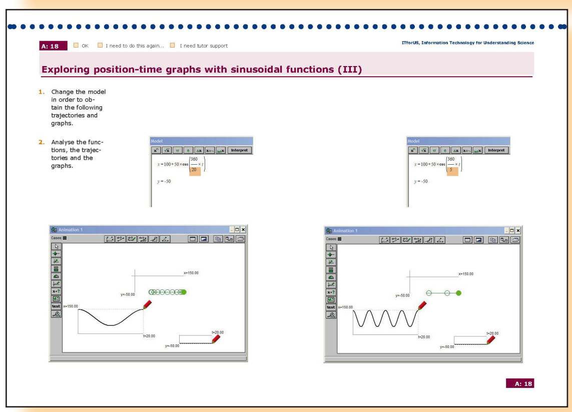
This set of activities assumes that the user has a basic knowledge of what is a **sinusoidal function** and that it can represent a **simple harmonic motion**.

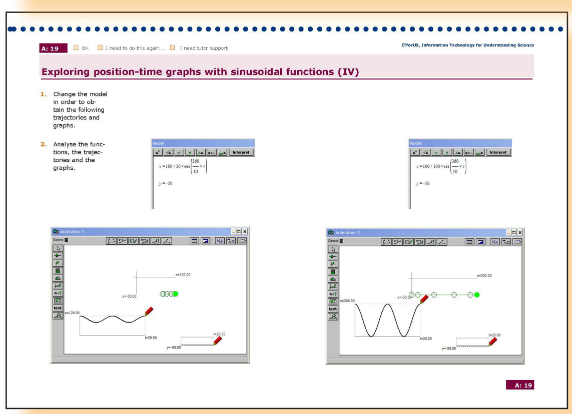
Basic instructions on how to:

1. define functions to describe coordinates x and y of a particle;
2. place the particle on the Animation Window with x as the horizontal coordinate and y as the vertical coordinate;
3. create a graph of x as a function of time t with adequate scales;
4. create a graph of y as a function of time t with adequate scales;
5. run the model and see the graphs of x and y in real time.





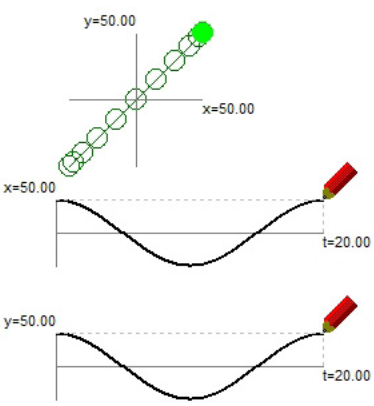




The particle is set to move only on one axis but the model could be easily changed to make the particle also move on both axis with sinusoidal functions. A **further exploration of this activity could include the circular or elliptic motion** or even simple harmonic motion on a line that makes an angle different from 0 or 90 degrees with the horizontal axis as shown on the example below.

$$x = 50 \times \cos\left(\frac{360}{20} \times t\right)$$

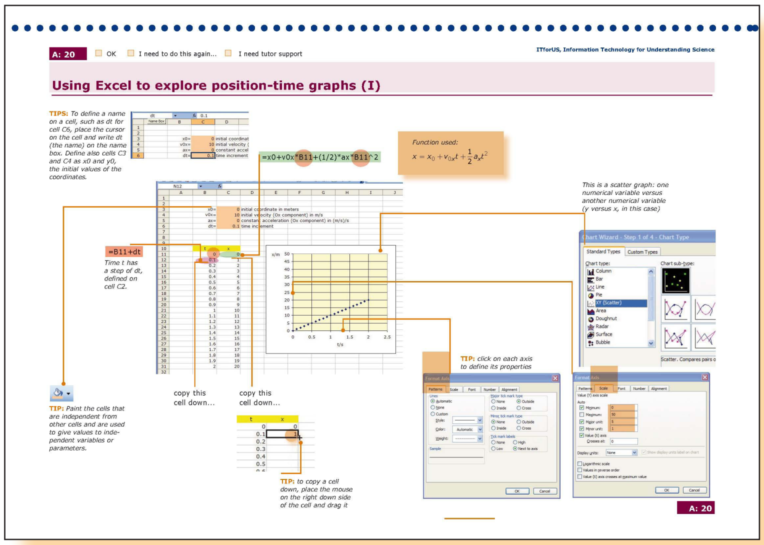
$$y = 50 \times \cos\left(\frac{360}{20} \times t\right)$$



Using Excel to explore position-time graphs

This set of activities shows how a spreadsheet like Excel can be used to make graphs of functions. Excel is an almost unavoidable tool not only in the general use of computers but also in Science and Mathematics.

A recommended sequence for introducing Excel to create a graph of a function is the following:



1 **define the parameters** (on the top left of the sheet..., see the *TIP* on how to define a name for a cell);

2 **define the time step** or time increment and label the cell;

3 create one column for the **independent variable** *t,* starting with the initial value (usually 0) and write on the following cell that its value is the previous value plus the time step;

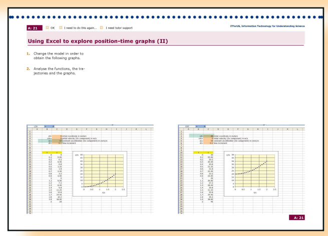
4 **copy** this cell to the cells below until the inpendent variable reaches the upper value you want;

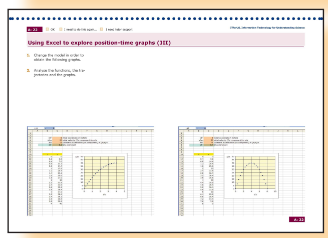
5 create another column for the **dependent variable** (in the current example, *x*, the horizontal coordinate);

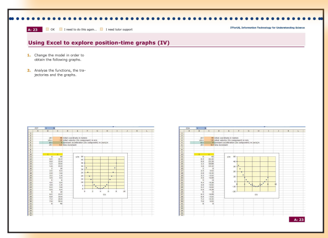
6 write on the first cell of this column the **expression that defines the function**, calling the parameters (using the cell names defined on the first step) and the current value of the independent variable on the same line of the first column;

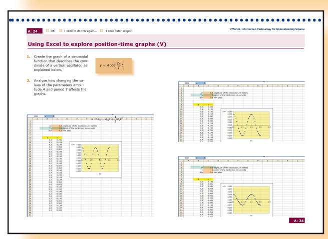
7 make a **graph** (“scatter graph”) after selecting the cells of the two columns;

8 define the **characteristics** of the graph (scales, lines, etc, clicking on it).









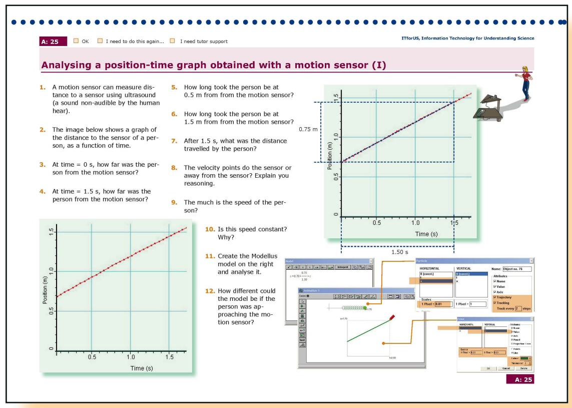
Using Excel to make graphs can help learners consolidate the essential features of a function because it **explicitely demands the user** to define parameters, the increment of the independent variable, the expression of the function, etc.

Another reason for using Excel is that since it has a completely different way of representing functions, when compared with educational software, it can help learners to focus on the concept of function instead of the specific features of how each software represents functions.

Analysing position-time graphs obtained with a motion sensor

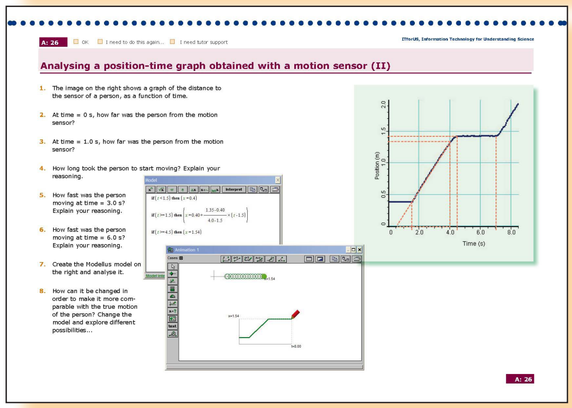
This set of activities shows how to use Modellus to make **models from graphs obtained with motion sensors**.

Motion sensor are very useful tools to help students relate the language of graphs and functions with their own motion or the motion of other objects.

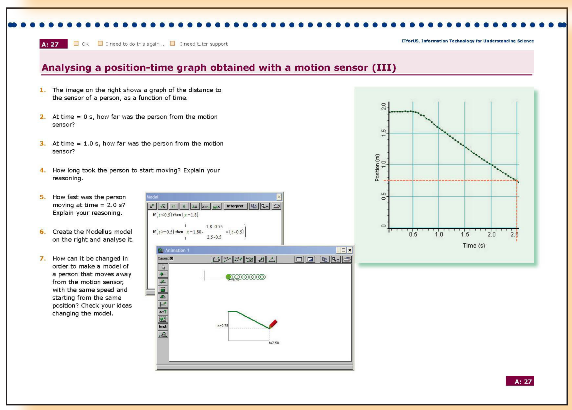


Examples I, II and III show motion with constant speed.

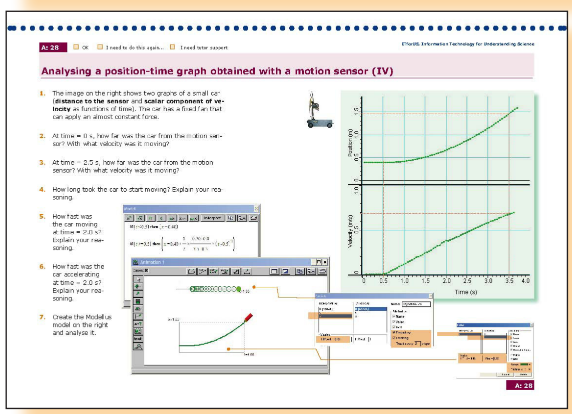
In the **first example**, the person was *moving away from the sensor* when the data logging system started measuring distance. **The speed was constant**: the model for the horizontal coordinate is a linear function with a slope that can be directly obtained from the graph.



In the **second example**, the person *was at rest* at a distance of 0.4 m from the sensor when the data logging system started measuring distance for about 1.5 s. Then the person *walked away* with a speed that can be measured by the slope of the position-time graph until 4.5 s, stopped for a while and then walked away again... To make a model with functions of this motion it is necessary to take into consideration the "time delay" of 1.5 s on the second step of the motion. In the end of the activity, students are invited to use a similar approach to the final step of the motion, after t = 7.0 s.



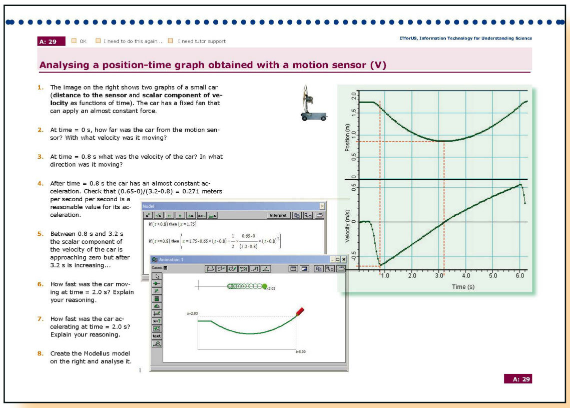
In the **third example**, the person *was at rest* at a distance of 0.4 m from the sensor when the data logging system started measuring distance for about 0.5 s. Then the person *walked to the sensor*...



Examples IV and V uses **motion with constant acceleration**.

In example IV, after 0,5 s, **a car with a fan accelerates away from the sensor**. The time-speed graph can be used to find the constant acceleration to create the quadratic function that describes the x coordinate of the motion.

In example V, after approximately 0,8 s, **a car with a fan is launched with a certain initial velocity to the sensor but accelerating away from the sensor**. The time-speed graph can be used to find initial velocity and the constant acceleration to create the quadratic function that describes the *x* coordinate of the motion.



Example VI uses a graph of the *y* coordinate of an **harmonic oscillator** (a small body suspended on a vertical spring). The period and the amplitude can be directly measured on the graph. Both sine and cosine functions can be used to describe the *y* coordinate (if time differ on a quarter of a period from one function to the other...!). Students can also change the Options... on the Control window to make the model using radians instead of degrees as unit for angles.

The model can be improved measuring the equilibrium point on the graph...